Grade 6

6.1. Core Content: *Multiplication and division of fractions and decimals (Numbers, Operations, Algebra)*

Students have done extensive work with fractions and decimals in previous grades and are now prepared to learn how to multiply and divide fractions and decimals with understanding. They can solve a wide variety of problems that involve the numbers they see every day—whole numbers, fractions, and decimals. By using approximations of fractions and decimals, students estimate computations and verify that their answers make sense.

Performance Expectations		***Explanatory Comments and Examples***
Students are expected to:		
6.1.A	Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =.	Examples: • List the numbers $2\frac{1}{3}$, $\frac{4}{5}$, 0.94, $\frac{5}{4}$, 1.1, and $\frac{43}{50}$ in increasing order, and then graph the numbers on the number line. • Compare each pair of numbers using <, >, or =. $\frac{4}{5} \square 1.2$ $\frac{7}{4} \square 1\frac{3}{4}$ $2\frac{7}{8} \square 2.5$
6.1.B	Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representation to the related equation.	This expectation addresses the conceptual meaning of multiplication and division of fractions and decimals. Students should be familiar with the use of visual representations like pictures (e.g., sketching the problem, grid paper) and physical objects (e.g., tangrams, cuisenaire rods). They should connect the visual representation to the corresponding equation. The procedures for multiplying fractions and decimals are addressed in 6.1.D and 6.1.E.
6.1.C	Estimate products and quotients of fractions and decimals.	Example: • $0.28 \div 0.96 \approx 0.3 \div 1$; $0.3 \div 1 = 0.3$ $0.24 \times 12.4 \approx \frac{1}{4} \times 12.4$; $\frac{1}{4} \times 12.4 = 3.1$ $\frac{3}{13} \times \frac{20}{41} \approx \frac{1}{4} \times \frac{1}{2}$; $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
6.1.D Fluently and accurately multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division with fractions.	Students should understand the inverse relationship between multiplication and division, developed in grade three and now extended to fractions. Students should work with different types of rational numbers, including whole numbers and mixed numbers, as they continue to expand their understanding of the set of rational numbers. Examples: • Multiply or divide. $\frac{4}{5} \times \frac{2}{3}$ $\quad 6 \div \frac{3}{8}$ $2\frac{1}{4} \times 3\frac{1}{2}$ $\quad 4\frac{1}{5} \div 1\frac{2}{3}$
6.1.E Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01, and 0.001.	This expectation extends what students know about the place value system and about multiplication and division and expands their set of mental math tools. As students work with multiplication by these powers of 10, they can gain an understanding of how numbers relate to each other based on their relative sizes. Example: • Mentally compute 0.01 x 435.
6.1.F Fluently and accurately multiply and divide non-negative decimals.	Students should understand the inverse relationship between multiplication and division, developed in grade three and now extended to decimals. Students should work with different types of decimals, including decimals greater than 1, decimals less than 1, and whole numbers, as they continue to expand their understanding of the set of rational numbers. Example: • Multiply or divide. 0.84×1.5 $\quad 2.04 \times 32$ $7.85 \div 0.32$ $\quad 17.28 \div 1.2$
6.1.G Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.	Examples: • Without doing any computation, list 74, 0.43×74, and $74 \div 0.85$ in increasing order and explain your reasoning. • Explain why $\frac{4}{0}$ is undefined.

Performance Expectations

Students are expected to:

6.1.H Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.

Explanatory Comments and Examples

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Every day has 24 hours. Ali sleeps 3/8 of the day. Dawson sleeps 1/3 of the day. Maddie sleeps 7.2 hours in a day. Who sleeps the longest? By how much?

Grade 6

6.2. Core Content: *Mathematical expressions and equations* *(Operations, Algebra)*

Students continue to develop their understanding of how letters are used to represent numbers in mathematics—an important foundation for algebraic thinking. Students use tables, words, numbers, graphs, and equations to describe simple linear relationships. They write and evaluate expressions and write and solve equations. By developing these algebraic skills at the middle school level, students will be able to make a smooth transition to high school mathematics.

Performance Expectations	***Explanatory Comments and Examples***
Students are expected to:	
6.2.A Write a mathematical expression or equation with variables to represent information in a table or given situation.	Examples: • What expression can be substituted for the question mark? <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>...</td><td>x</td></tr><tr><td>y</td><td>2.5</td><td>5</td><td>7.5</td><td>10</td><td>...</td><td>?</td></tr></table> • A t-shirt printing company charges $7 for each t-shirt it prints. Write an equation that represents the total cost, *c*, for ordering a specific quantity, *t*, of these t-shirts.
6.2.B Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.	Example: • Mikayla and her sister are making beaded bracelets to sell at a school craft fair. They can make two bracelets every 30 minutes. Draw a graph that represents the number of bracelets the girls will have made at any point during the 6 hours they work.
6.2.C Evaluate mathematical expressions when the value for each variable is given.	Examples: • Evaluate $2s + 5t$ when $s = 3.4$ and $t = 1.8$. • Evaluate $\frac{2}{3}x - 14$ when $x = 60$.
6.2.D Apply the commutative, associative, and distributive properties, and use the order of operations to evaluate mathematical expressions.	Examples: • Simplify $6\left(\frac{1}{2}+\frac{1}{3}\right)$, with and without the use of the distributive property. • Evaluate $b - 3(2a - 7)$ when $a = 5.4$ and $b = 31.7$.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
6.2.E Solve one-step equations and verify solutions.	Students solve equations using number sense, physical objects (e.g., balance scales), pictures, or properties of equality. Example: • Solve for the variable in each equation below. $112 = 7a$ $1.4y = 42$ $2\frac{1}{2} = b + \frac{1}{3}$ $\frac{y}{45} = \frac{7}{15}$
6.2.F Solve word problems using mathematical expressions and equations and verify solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Zane and his friends drove across the United States at an average speed of 55 mph. Write expressions to show how far they traveled in 12 hours, in 18 hours, and in *n* hours. How long did it take them to drive 1,430 miles? Verify your solution.

Grade 6

6.3. Core Content: *Ratios, rates, and percents* *(Numbers, Operations, Geometry/Measurement, Algebra, Data/Statistics/Probability)*

Students extend their knowledge of fractions to develop an understanding of what a ratio is and how it relates to a rate and a percent. Fractions, ratios, rates, and percents appear daily in the media and in everyday calculations like determining the sale price at a retail store or figuring out gas mileage. Students solve a variety of problems related to such situations. A solid understanding of ratios and rates is important for work involving proportional relationships in grade seven.

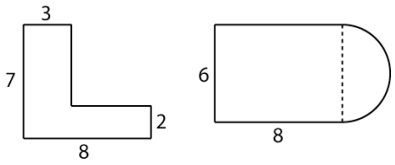
Performance Expectations	***Explanatory Comments and Examples***
Students are expected to:	
6.3.A Identify and write ratios as comparisons of part-to-part and part-to-whole relationships.	Example: • If there are 10 boys and 12 girls in a class, what is the ratio of boys to girls? What is the ratio of the number of boys to the total number of students in the class?
6.3.B Write ratios to represent a variety of rates.	Example: • Julio drove his car 579 miles and used 15 gallons of gasoline. How many miles per gallon did his car get during the trip? Explain your answer.
6.3.C Represent percents visually and numerically, and convert between the fractional, decimal, and percent representations of a number.	In addition to general translations among these representations, this expectation includes the quick recall of equivalent forms of common fractions (with denominators like 2, 3, 4, 5, 8, and 10), decimals, and percents. It also includes the understanding that a fraction represents division, an important conceptual background for writing fractions as decimals. Examples: • Represent $\frac{75}{100}$ as a percent using numbers, a picture, and a circle graph. • Represent 40% as a fraction and as a decimal. • Write $\frac{13}{16}$ as a decimal and as a percent.

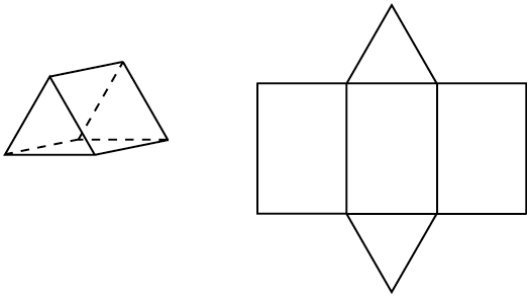
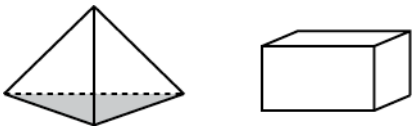
Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
6.3.D Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Examples: • An item is advertised as being 25% off the regular price. If the sale price is $42, what was the original regular price? Verify your solution. • Sally had a business meeting in a city 100 miles away. In the morning, she drove an average speed of 60 miles per hour, but in the evening when she returned, she averaged only 40 miles per hour. How much longer did the evening trip take than the morning trip? Explain your reasoning.
6.3.E Identify the ratio of the circumference to the diameter of a circle as the constant π, and recognize $\frac{22}{7}$ and 3.14 as common approximations of π.	Example: • Measure the diameter and circumference of several circular objects. Divide each circumference by its diameter. What do you notice about the results?
6.3.F Determine the experimental probability of a simple event using data collected in an experiment.	The term *experimental probability* refers here to the relative frequency that was observed in an experiment. Example: • Tim is checking the apples in his orchard for worms. Selecting apples at random, he finds 9 apples with worms and 63 apples without worms. What is the experimental probability that a given apple from his orchard has a worm in it?
6.3.G Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal from 0 to 1 or as a percent from 0 to 100.	Example: • A bag contains 4 green marbles, 6 red marbles, and 10 blue marbles. If one marble is drawn randomly from the bag, what is the probability it will be red? What is the probability that it will not be red?

Grade 6

6.4. Core Content: *Two- and three-dimensional figures* *(Geometry/Measurement, Algebra)*

Students extend what they know about area and perimeter to more complex two-dimensional figures, including circles. They find the surface area and volume of simple three-dimensional figures. As they learn about these important concepts, students can solve problems involving more complex figures than in earlier grades and use geometry to deal with a wider range of situations. These fundamental skills of geometry and measurement are increasingly called for in the workplace and they lead to a more formal study of geometry in high school.

Performance Expectations	***Explanatory Comments and Examples***
Students are expected to:	
6.4.A Determine the circumference and area of circles.	Examples: • Determine the area of a circle with a diameter of 12 inches. • Determine the circumference of a circle with a radius of 32 centimeters.
6.4.B Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.	Although students have worked with various quadrilaterals in the past, this expectation includes other quadrilaterals such as trapezoids or irregular quadrilaterals, as well as any other composite figure that can be divided into figures for which students have calculated areas before. Example: • Determine the area and perimeter of each of the following figures, assuming that the dimensions on the figures are in feet. The curved portion of the second figure is a semi-circle. 
6.4.C Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Captain Jenkins determined that the distance around a circular island is 44 miles. What is the distance from the shore to the buried treasure in the center of the island? What is the area of the island?

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
6.4.D Recognize and draw two-dimensional representations of three-dimensional figures.	The net of a rectangular prism consists of six rectangles that can then be folded to make the prism. The net of a cylinder consists of two circles and a rectangle. Example: • 
6.4.E Determine the surface area and volume of rectangular prisms using appropriate formulas and explain why the formulas work.	Students may determine surface area by calculating the area of the faces and adding the results.
6.4.F Determine the surface area of a pyramid.	
6.4.G Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.	Prisms and pyramids are the focus at this level. Examples: • How many pairs of parallel faces does each polyhedron have? Explain your answer.  • What type of polyhedron has two parallel triangular faces and three non-parallel rectangular faces?

Grade 6

6.5. Additional Key Content *(Numbers, Operations)*

Students extend their mental math skills now that they have learned all of the operations—addition, subtraction, multiplication, and division—with whole numbers, fractions, and decimals. Students continue to expand their understanding of our number system as they are introduced to negative numbers for describing positions or quantities below zero. These numbers are a critical foundation for algebra, and students will learn how to add, subtract, multiply, and divide positive and negative numbers in seventh grade as further preparation for algebraic study.

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

6.5.A Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.

Examples:

- John wants to find the total number of hours he worked this week. Use his time card below to find the total.

Days	Monday	Tuesday	Wednesday	Thursday	Friday
Days	$4\frac{1}{4}$	3	$6\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{1}{2}$

- What is the total cost for items priced at $25.99 and $32.95? (A student may think of something like 25.99 + 32.95 = (26 + 33) – 0.06 = 58.94.)

6.5.B Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.

Contexts could include elevation, temperature, or debt, among others.

6.5.C Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.

Examples:

- Compare each pair of numbers using <, >, or =.

  –11 ☐ –14

  –7 ☐ 4

  –101 ☐ –94

Grade 6

6.6. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven.

Performance Expectations

Students are expected to:

6.6.A Analyze a problem situation to determine the question(s) to be answered.

6.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

6.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

6.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

6.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

6.6.F Apply a previously used problem-solving strategy in a new context.

6.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

6.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- As part of her exercise routine, Carmen jogs twice around the perimeter of a square park that measures $\frac{5}{8}$ mile on each side. On Monday, she started at one corner of the park and jogged $\frac{2}{3}$ of the way around in 17 minutes before stopping at a small pond in the park to feed some ducks. How far had Carmen run when she reached the pond? What percent of her planned total distance had Carmen completed when she stopped to feed the ducks? If it took Carmen 17 minutes to jog to the point where she stopped, assuming that she continued running in the same direction at the same pace and did not stop again, how long would it have taken her to get back to her starting point? Explain your answers.

- At Springhill Elementary School's annual fair, Vanessa is playing a game called "Find the Key." A key is randomly placed somewhere in one of the rooms shown on the map below. (The key cannot be placed in the hallway.)

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

To win the game, Vanessa must correctly guess the room where the key is placed. Use what you know about the sizes of the rooms to determine the probability that the key is placed in the gym, the office, the café, the book closet, or the library. Write each probability as a simplified fraction, a decimal, and a percent. Which room should Vanessa select in order to have the best chance of winning? Justify the solution.